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Hemodynamical Modeling in Two Phase Pulmonary Circulation during Inflammation

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Abstract
This study focuses on a Non-Newtonian mathematical model of two phase in human pulmonary artery; Keeping in view the nature of blood flow during lung cancer. Non-Newtonian power law model applied which transformed into bio-fluid mechanical set up. For the purpose, blood has been assumed to be constituted by plasma and red blood cells which is realistic so for. Clinically evaluation of clinical data collected in a hospital is also presented graphically. This study aims at providing the considerable role of red blood cells in two phase blood flow. The clinical data based on empirical ground is analyzed with the help of mathematical interpretation. In present study overall presentation is in tensorial form and the solution technique adopted analytical as well as numerical. Fluctuation of blood flow determined explores the role of hematocrit, which is very important in view of the need of medical science at the present time.
Keywords:

Introduction
The most important feature of the circulation in human body; if a given volume of blood is pumped from the heart the same will return on the heart after passing through the different sub divisions of the circulatory system. The term stenosis denoted the narrowing of the artery due to the development of arteriosclerotic plaques or other types of abnormal tissue development. As the growth projects in to the lumen (cavity) of the artery, blood flow is obstructed. The obstruction may damage the internal cells of the wall and may lead to further growth of stenosis. Thus there is a coupling between the growth of a stenosis and the flow of blood in the artery since each affects the others [Upadhyay V., 2000].

Because according to Upadhyay V. and Pandey P. N., whenever the hematocrit increases, the effective viscosity of blood flowing in the arteries remote from the heart depends upon the strain rate. In this condition, the blood flow becomes non-Newtonian. When strain rate is in 5 to 200 per second, the power law $\tau' = \eta_m e^n$, Where, $0.68 \leq n \leq 0.80$, holds good for blood flow. So we have applied non Newtonian power law model for in this condition the constitutive equation of blood is as follows.

$$\tau^{ij} = -pg^{ij} + \eta_m (e^{ij})^n = -pg^{ij} + \tau'^{ij}$$

Where, τ^{ij} is stress tensor and τ'^{ij} is shearing stress tensor.

I. Real model
Choice of frame of reference:

According to Upadhyay V. and Pandey P. N. 2000, we have secured a frame of reference is selected for mathematical modeling of two-phase blood flow of the state of a moving blood· It is experiential in view the difficulty and generality of the problem of blood flow, selected generalized three-dimensional orthogonal curvilinear coordinate system, briefly prescribed as E_3 , called as 3-dim Euclidean space. It is interpreted the quantities related to blood flow in

a tensorial form which is comparatively more realistic, the biophysical laws thus expressed completely hold good in any coordinate system, which is the compulsion for the reliability of the law. Now, let the co-ordinate axes be OX^i, O denotes source and superscript $i = 1, 2, 3$, let X^i be the co-ordinates of any point P in space.

Parameterization of Bio-physical Problem:

The mathematical description of the state if a moving blood was affected by means of functions which give the distribution of the blood velocity $v^k = v^k\ (X^i, t)$, K=1, 2, 3, and of any two thermodynamic quantities pertaining to the blood, for instance the pressure $P = P\ (X^i, t)$, and the density $\rho = \rho\ (X^i, t)$. As was well known, all the thermodynamic quantities are determined by the values of any two of them, together with the equate of state. Hence, if we are given five quantities, namely the three components of velocity v^k, the pressure P and the density ρ, the state of moving blood was completely determined.

All these quantities are functions of the co-ordinates $X^i, X = 1, 2, 3$ and of the time t. It emphasized that $v^k\ (X^i, t)$ was the velocity of the blood at a given point X^i in space and at a given t. The same remarks apply to P and ρ Blood was a mixture of fluid [Upadhyay, 2000].

Hypothesis of two phase blood volume:

Blood has always held a special position in human though. The quantity of blood in the body is substantial, making up about 7% of the total body weight. Blood function in the transport of blood, oxygen, waste material and hormones in the regulation of temperature and in the control of disease [Upadhyay V., 2000]. According to Bessonov *et al*., 2016; "The human blood is a concentrated suspension of several formed cellular elements. The human blood cells volume more than 99% of all blood cells and total volume concentration of leukocytes and thrombocytes is only about 1%". Which is ignorable; so we have selected two phases where one phase-plasma and another is red blood cells phase. Plasma is a liquid, containing semi permeable packages of RBCs. The behavior of blood is about Newtonian at the high shear rate, while at law shear rate the blood exhibits yield stress and non-Newtonian behavior. The flow of blood is precious by the presence of blood cells. This effect is directly proportional to the volume taken by blood cells. Let the volume portion covered by blood cells in unit volume be X, X is replaced by H /100, where H is the hematocrit the volume percentage of blood cells. The hematocrit is normally about three times the hemoglobin concentration 'reported as grams per deciliter' (Berkow, 1997). Then the volume portion covered by the plasma will be (1-X). The mass ratio of cells to plasma is r is given by:

$$r = \frac{X\rho_c}{(1-X)\rho_p}.$$

Where ρ_c and ρ_p are densities of blood cells and plasma respectivily.

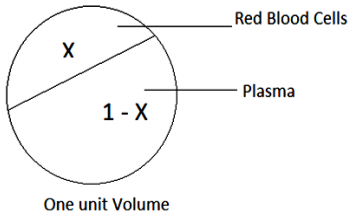


Fig: 1. Blood one unit

Usually, this mass ratio is not a steady. Even then this may be supposed to constant at present steady [Upadhyay, 2000].

II. Mathematical formation

We take the blood flow in pulmonary arteries remote from heart to be non-Newtonian power law. It is also supposed to be steady. The stenosis is taken to be axially-symmetric whose surface is given by the following equation.

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \frac{\cos\pi z}{Z_0}\right), -Z_0 \le Z \le Z_0$$

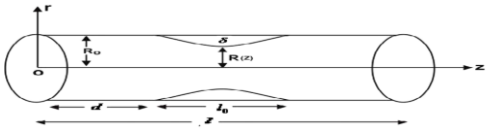


Fig: 2. Pulmonary Artery stenosis

We shall assume further that $\frac{\delta}{R_0} <<<$

$1\ and\ \mathrm{R_e}\frac{\delta}{\mathrm{R_0}} <<< 1$

Where, $\mathrm{R_e}$ the Reynolds number fluid flow, now we write the basic equations of blood flow in tensorial form as follows -

Equation of continuity:

$$\frac{1}{\sqrt{\mathrm{g}}(\sqrt{\mathrm{g}}v^{\mathrm{i}})_{,\mathrm{i}}} = 0 \qquad (3.1)$$

Equation of motion:

$$\rho_{\mathrm{m}}\frac{\partial v^{\mathrm{i}}}{\partial \mathrm{t}} + \left(\rho_{\mathrm{m}}v^{\mathrm{i}}\right)v^{\mathrm{i}}_{,\mathrm{j}} = \tau^{\mathrm{ij}}_{,\mathrm{j}} \qquad (3.2)$$

While the constitutive equations for the power law non-Newtonian blood flow is as follows:

$$\tau^{\mathrm{ij}} = -\mathrm{pg}^{\mathrm{ij}} + \eta_{\mathrm{m}}\left(\mathrm{e}^{\mathrm{ij}}\right) + \tau'^{\mathrm{ij}} \qquad (3.3)$$

Where all the symbols have their usual meaning.

By caring out the an order of magnitude analysis on these basic equation of flow transferred into cylindrical form, it can be observed that the radial velocity can be neglected in relation to axial velocity v which is determined by

$$0 = -\frac{\partial \mathrm{p}}{\partial \mathrm{r}} \qquad (3.4)$$

$$0 = -\left(\frac{\partial \mathrm{p}}{\partial \mathrm{z}}\right) + \left(\frac{\eta_{\mathrm{m}}}{\mathrm{r}}\right)\frac{\partial}{\partial \mathrm{r}}\left\{\mathrm{r}\left(\frac{\partial v}{\partial \mathrm{r}}\right)^{\mathrm{n}}\right\} \qquad (3.5)$$

The no slip condition on the stenosis surface is given by

$$v = 0 \text{ at } r = R(z), -Z_0 \le Z \le Z_0 \qquad (3.6)$$

$v = 0$, at $r = R_0 |\mathrm{Z}| \ge \mathrm{Z}_0$.

III. Solution

We know that pressure gradient $\mathrm{P} = -\frac{\partial \mathrm{p}}{\partial \mathrm{z}}$ and axial velocity are function of z also.

Hence the equation (3.5) and (3.6) can be combined into the following equations

$$-\mathrm{P(z)} = \left(\frac{\eta_{\mathrm{m}}}{\mathrm{r}}\right)\frac{\partial}{\partial \mathrm{r}}\left\{\mathrm{r}\left(\frac{\partial v}{\partial \mathrm{r}}\right)^{\mathrm{n}}\right\} \qquad (4.1)$$

Integrating (4.1) get- $\mathrm{r}\left(\frac{\partial v}{\partial \mathrm{r}}\right)^{\mathrm{n}} = -\frac{\mathrm{P(z)r}^2}{2\eta_{\mathrm{m}}} + \mathrm{A(z)}$ (4.2)

Applying the boundary condition at r = 0, $\frac{\partial v}{\partial \mathrm{r}} = 0$, we obtain the value of arbitrary constant A(z) = 0

The equation transform into following form

$$\mathrm{r}\left(\frac{\partial v}{\partial \mathrm{r}}\right)^{\mathrm{n}} = -\frac{\mathrm{P(z)r}^2}{2\eta_{\mathrm{m}}} - \left(\frac{\partial v}{\partial \mathrm{r}}\right) = \left(\frac{\mathrm{P(z)r}}{2\eta_{\mathrm{m}}}\right)^{\frac{1}{\mathrm{n}}} \qquad (4.3)$$

Integrating this equation we get -

$$v = -\left(\frac{\mathrm{P(z)r}}{2\eta_{\mathrm{m}}}\right)^{\frac{1}{\mathrm{n}}}\frac{\mathrm{n}}{\mathrm{n+1}}\mathrm{r}^{\left(\frac{1}{\mathrm{n}}+1\right)} + \mathrm{B(z)} \qquad (4.4)$$

Applying the boundary condition (3.6) we get the value arbitrary constant

$$B(z) = \left(\frac{\mathrm{P(z)}}{2\eta_{\mathrm{m}}}\right)\frac{\mathrm{n}}{\mathrm{n+1}}\mathrm{R(z)}^{\frac{1}{\mathrm{n}}+1}$$

Hence the equation (4.4) reduces in the form of

$$v = \frac{\mathrm{n}}{\mathrm{n+1}}\left(\frac{\mathrm{P(z)r}}{2\eta_{\mathrm{m}}}\right)^{\frac{1}{\mathrm{n}}}\left(\mathrm{R(z)}^{\frac{1}{\mathrm{n}}+1} - \mathrm{r}^{\frac{1}{\mathrm{n}}+1}\right) \qquad (4.5)$$

The above formula gives the velocity of blood flow passing through stenosis.

IV. Result and discussion:

The total flow-flux of blood through the transverse section of the arteries is given as

$$\mathrm{Q} = \int_0^{\mathrm{R}} \mathrm{v}.\,2\pi \mathrm{r}\,\mathrm{dr}$$

$$= \int_0^{\mathrm{R(z)}} \frac{\mathrm{n}}{\mathrm{n+1}}\left(\frac{\mathrm{P(z)r}}{2\eta_{\mathrm{m}}}\right)^{\frac{1}{\mathrm{n}}}\left(\mathrm{R(z)}^{\frac{1}{\mathrm{n}}+1} - \mathrm{r}^{\frac{1}{\mathrm{n}}+1}\right) 2\pi \mathrm{dr}$$

$$= \left(\frac{\pi \mathrm{n}}{3\mathrm{n}+1}\right) \times \left[\frac{\mathrm{P(z)}}{2\eta_{\mathrm{m}}}\right]^{\frac{1}{\mathrm{n}}} [\mathrm{R(z)}]^{\frac{1}{\mathrm{n}}+3} \qquad (5.1)$$

V. Bio-physical interpretation:

We have collected 20 % stenosed pulmonary artery of lung cancer patient; we know that the average human pulmonary blood flow flax $(Q) = 0.00708333\ m^3/\mathrm{sec\ ond}$, approximately conman radius of pulmonary artery $(\mathrm{R_0}) =$ 1.5 cm or 0.015m, according to Gustafson, Daniel R. (1980), $\eta_{\mathrm{p}} = 0.0013$ pascal second , according to Glenn Elert (2010), $\eta_{\mathrm{m}} = 0.0271$ pascal second . Approximately pulmonary artery length $(z_f - z_i) = 5\ cm$ or o.05m and we get-

δ = 20% of radius of pulmonary artery

$= 0.015 \times .2 \Rightarrow 0.003\ m.$ and $R(z) = \mathrm{R0} - \delta = 0.015 - 0.003 = 0.012$ m.

Examination of hematocrit v/s blood pressure in during lung cancer with respect to clinical data for patient- S (Female), 45 years old.

Table (1) Hemoglobin & blood pressures in clinical data

Date	HB(Hemoglobin (gram/dl)	Hematocrit In $(3 \times HB)$ (kg/l)	Blood Pressure (BP) in (mmhg)	Arteries Pressure Drop In Pascal-second $\left(\frac{S+D}{2}\right) - S$
9/8/2013	11.37	0.032179245	130/80	-3328.3
23/9/2013	11.18	0.031641509	110/70	-2662.64
28/10/2013	11.0	0.031132075	130/70	-3993.96
12/11/2013	10.8	0.030566038	100/70	-1996.98
26/11/2013	10.7	0.030283019	110/80	-1996.98
8/12/2013	10.5	0.029716981	100/60	-2662.64

In according to used clinical data (Table: 1) (Hematocrit) $H = 0.031132075$ and Pressure drop $(P_f - P_i) = 3993.96\ Pascal\ second.$

$$P(z) = \frac{P_f - P_i}{z_f - z_i}$$

And by using relation

$\eta_m = \eta_c X + \eta_p (1 - X)$ ..... (A)

Where, $X = \frac{H}{100}$, and we get η_c

$\eta_m = \eta_c X + \eta_p (1 - X) \Rightarrow 0.0271 = \eta_c(0.00031132075) + 0.0013(0.999688679)$

$\eta_c = 82.87402853$

Pascal second

Again using (A) relation and change in to the hematocrit-

$\eta_m = \eta_c X + \eta_p (1 - X)$

$\Rightarrow \eta_m = 0.8287402853H + 0.001299$..... (M)

Now flow flax is given by from equation (5.1)

$Q = \frac{\pi n}{3n+1} \times \left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}} (R(z))^{\frac{1}{n}+3}$ ..... (B)

$0.00708333 =$

$\left(\frac{\pi n}{3n+1}\right) \times \left(\frac{3993.96}{2\times0.0271\times0.05}\right)^{\frac{1}{n}} (0.012)^{\frac{1}{n}+3}$

$\left(\frac{0.00708333}{(0.012)^3\pi}\right) = \left(\frac{n}{3n+1}\right) \times \left(\frac{3993.96}{2\times0.0271\times0.05}\right)^{\frac{1}{n}} (0.012)^{\frac{1}{n}}$

$1305.461562 = \left(\frac{n}{3n+1}\right) (17685.43173)^{\frac{1}{n}}$

On solving above equation by used trial and error method and we get n = 0.878741

Now apply n = 0.878741, again using equation (B) and get Δp

$0.00708333 =$

$\left(\frac{0.878741\times3.14}{(3\times0.878741)+1}\right)\left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}} (0.012)^{\frac{1}{0.878741}} \times (0.012)^3$

$\left(\frac{0.00708333}{\left(\frac{0.878741\times3.14}{(3\times0.878741)+1}\right)\times(0.012)^{\frac{1}{0.878741}}\times(0.012)^3}\right) = \left(\frac{P(z)}{2\eta_m}\right)^{\frac{1}{n}}$

$\left(\frac{0.00708333}{\left(\frac{0.878741\times3.14}{(3\times0.878741)+1}\right)\times(0.012)^{\frac{1}{0.878741}}\times(0.012)^3}\right)^n \left(\frac{\Delta p}{2\eta_m\times(z_f - z_i)}\right)$

$\left(\frac{0.00708333}{\left(\frac{0.878741\times3.14}{(3\times0.878741)+1}\right)\times(0.012)^{\frac{1}{0.878741}}\times(0.012)^3}\right)^n \frac{1}{2\eta_m}\left(\frac{\Delta p}{(z_f - z_i)}\right)$

$\Delta p = 2\eta_m \times (z_f - z_i) \times (263303.0569)^{0.878741}$

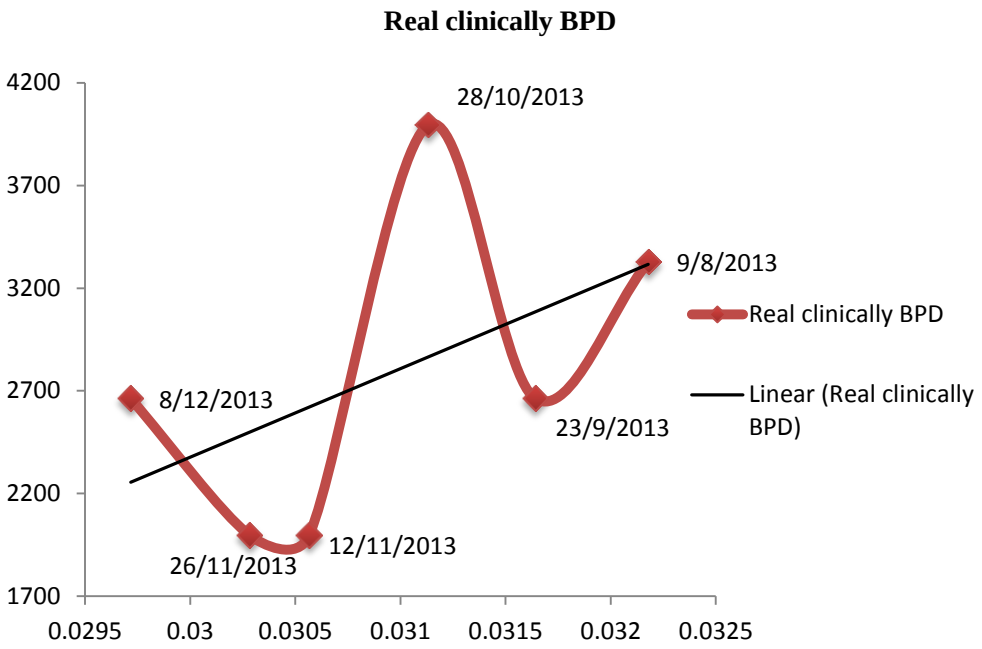
$\Delta p = \eta_m(5796.646803)$, (using equation (M) and getΔp-)

$\Delta p = (5796.646803)\ (\ 0.8287402853H + 0.001299595\ )$

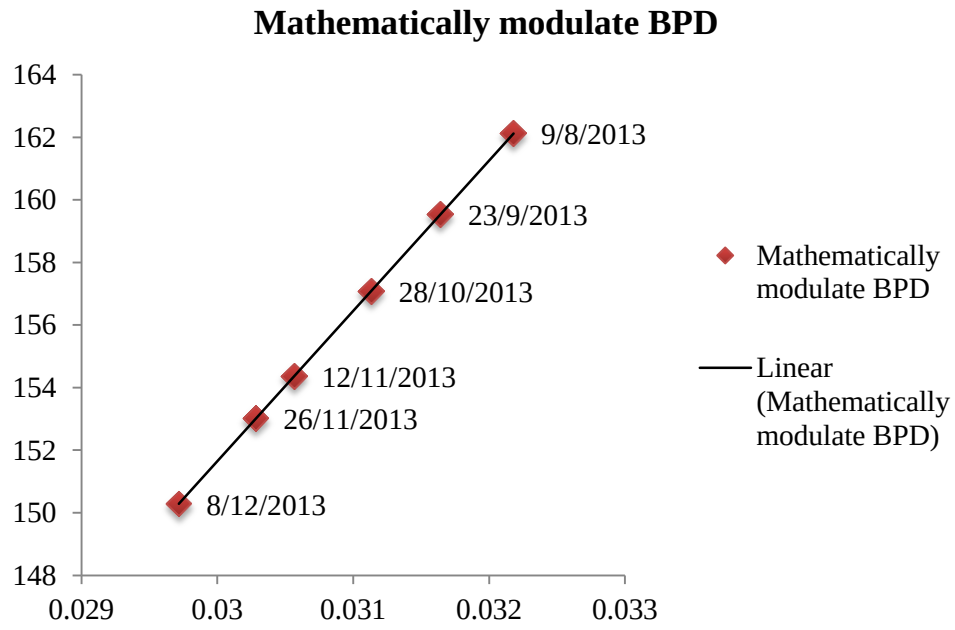
$\Delta p = (4803.914725H + 7.53329484)$

Table (2) Blood pressures drop v/s Hematocrit in Clinical data

Date	Hematocrit in $(3 \times HB)$ (kg/m^3)	BPD (Blood Pressure drop) In Pascal-seco
9/8/2013	0.032179245	162.1196438
23/9/2013	0.031641509	159.5364059
28/10/2013	0.031132075	157.0891284
12/11/2013	0.030566038	154.3699349
26/11/2013	0.030283019	153.0103357
8/12/2013	0.029716981	150.2911375



Graph a (table- 1) relation between real clinically blood pressure drop and hematocrit. $BPD = \left(\frac{S+D}{2} - S\right)$, Where S = Systolic blood pressure and D =



Graph b (table: 2) relations between mathematically modulated blood pressure drop v/s hematocrit $\Delta p = (4803.914725H + 7.533294843$. Where Δp is denoted by Relation between blood pressure

Observation:

Graph (a) & (b) shows the relationship between blood pressure drop and hematocrit of lung cancer patient for various dates. Graph (a) shows that these 6 different dates were observed minimum about 1996.98 on dated 12/11/2013 and maximum value obtain 3993.96 on dated 28/10/2013 (BDP). The value from 0.029716981 to 0.030566038 via 0.030283019 of hematocrit value, the blood pressure drop shows slightly down convex in increasing sense and the value from 0.030566038 to 0.031641509 via 0.031132075 of hematocrit value, the blood pressure drop shows down convex in decreasing sense but the value from 0.031132075 to 0.032179245 via 0.031641509 hematocrit value, the blood pressure drop shows down convex in increasing sense. Graph (b) shows that these 6 different dates were observed minimum about 150.2911375 on dated 8/12/2013 and maximum value obtains 162.1196438 on dated 9/8/2013 (BPD). At the value from 0.032179245 to 0.029716981 via 0.031641509, 0.031132075, 0.030566038 & 0.030283019 of hematocrit value, the blood pressure drop straightly decreases on dated 9/8/2013 to 8/12/2013 via 23/9/2013, 28/10/2013, 12/11/2013 & 26/11/2013.

Conclusion:

Mathematically investigated and concluded of the figure 1; graph b (table 2) shows from 9/8/2013 to 8/12/2013 via 23/9/2013, 28/10/2013, 12/11/2013 & 26/11/2013 decreasing sence. When blood pressure drop is increased (from 8/12/2013 to 9/8/2013) then we cannot suggest for operation but when blood pressure drop is decreased we suggest for successful operation. Between 12/11/2013 & 26/11/2013 to 8/12/2013 successful operation is suggested otherwise not subject to the condition that the clinical data is collected in the duration of declared operation

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